Prediction of seawater intrusion to coastal aquifers based on non-dimensional diagrams

Ch. Doulgeris[1*], E. Tziritis[1], V. Pisinaras[1], A. Panagopoulos[1] and C. Külls[2]

[1]Soil & Water Resources Institute, Hellenic Agricultural Organisation "DEMETER", Sindos, Central Macedonia, Greece
[2]Laboratory for Hydrology and International Water Management, University of Applied Sciences, Lübeck, Germany

*corresponding author:
ch.doulgeris@swri.gr

1. Introduction

Coastal aquifers are a valuable source of freshwater, which is particularly at risk due to their proximity to sea. Numerical models are useful tools for evaluating the effectiveness of measures that can be taken to protect or restore aquifers from seawater intrusion, such as, pumping control and re-allocation, promotion of natural and artificial recharge, etc. However, a quick and reliable assessment of seawater intrusion is invaluable to predict in time the potential risk facing a coastal aquifer.

Herein,

- we consider the — classical seawater intrusion — Henry problem, which concerns the evolution of seawater in a homogeneous and isotropic confined aquifer that is recharged on one side (e.g. right) by a freshwater influx and is exposed on the other side to a body of seawater (Fig. 1),
- we construct non-dimensional diagrams to predict seawater intrusion according to numerical simulations based on the density-driven flow approach

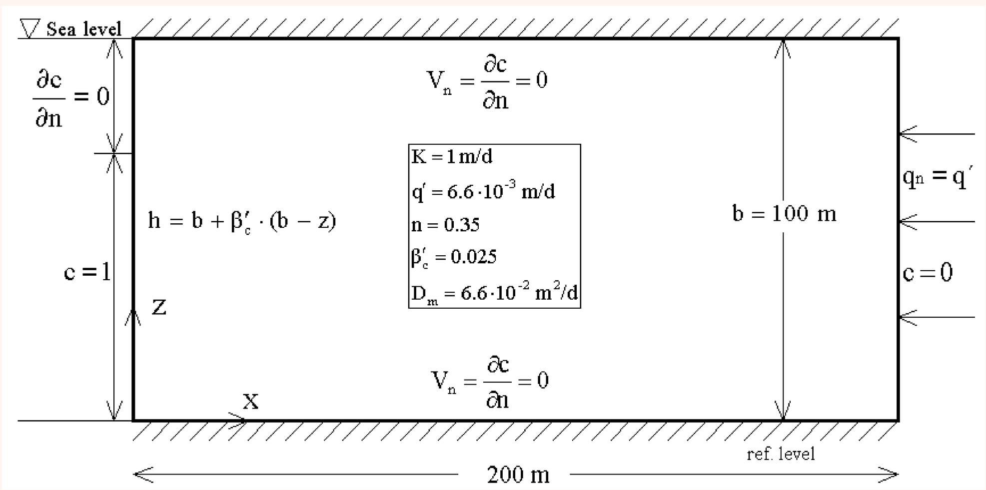

Fig.1. *The 2D Henry's problem – Hydraulic parameters and boundary conditions*

2. Numerical model

The numerical simulations for the construction of the non-dimensional diagrams were performed using a three-dimensional finite element model (Doulgeris, 2005; Doulgeris and Zissis, 2014). The governing equations of the model are:

<u>Flow equation</u>

$$\nabla \cdot \left[\boldsymbol{K} \cdot (\nabla h + \beta'_c c \nabla z) \right] = S_S \frac{\partial h}{\partial t} + n\,\beta'_c \frac{\partial c}{\partial t} - \frac{\rho_R}{\rho_0} Q_R + \frac{\rho}{\rho_0} Q_P \qquad (1)$$

<u>Transport equation</u>

$$\nabla \cdot (\boldsymbol{D} \cdot \nabla c) = W_c \left(\frac{\partial c}{\partial t} + \mathbf{V} \cdot \nabla c\right) - W'_c Q_R \qquad (2)$$

where $W_c = \left[1 - \frac{\rho_0}{\rho} \beta'_c \left(\frac{c_0}{c_s - c_0} + c\right)\right] n$ (3a) $\quad W'_c = \frac{c_R}{c_s - c_0} - \frac{\rho_R}{\rho}\left(\frac{c_0}{c_s - c_0} + c\right)$ (3b)

<u>Darcy's Law</u>

$$\mathbf{V} = -\frac{\rho_0}{\rho\, n} \boldsymbol{K} \cdot (\nabla h + \beta'_c c \nabla z) \qquad (4)$$

<u>Equation of state</u>

$$\rho = \rho_0 (1 + \beta'_c c) \qquad (5a) \qquad\qquad \beta'_c = \frac{\rho_s - \rho_0}{\rho_0} \qquad (5b)$$

where
h $[=p/(\rho_0 g) + z]$: the freshwater hydraulic head
c $[=(c'-c_0)/(c_s-c_0)]$: the dimensionless solute concentration
β'_c $[=(\rho_s-\rho_0)/\rho_0]$: dimensionless coefficient of density difference

3. Non-dimensional analysis

The governing equations of the numerical model (eq. 1 and 2) could be written, under certain conditions, in non-dimensional form. Thus, assuming that:

- the aquifer layer is homogeneous and isotropic, with horizontal base and constant thickness
- the aquifer layer is uniformly recharged with freshwater
- the hydrodynamic dispersion depends solely on molecular diffusion, i.e. $D_h=D_m$

we may define the non-dimensional variables and the non-dimensional groundwater flow velocities:

$$x' = \frac{x}{b} \quad z' = \frac{z}{b} \quad h' = \frac{h}{b} \quad t' = \frac{K_o}{n \cdot b} t \qquad (6a,b,c,d) \qquad Q_x = -\frac{bK^0}{nD_m}\frac{\partial h'}{\partial x'} \quad Q_z = -\frac{bK^0}{nD_m}\left(\frac{\partial h'}{\partial z'} + \beta'_c c\right) \qquad (7a,b)$$

which we substitute in the two dimensional form of equations 1 and 2:

$$\frac{\partial}{\partial x'}\left[(1+\beta'_c c)\frac{\partial h'}{\partial x'}\right] + \frac{\partial}{\partial z'}\left[(1+\beta'_c c)\left(\frac{\partial h'}{\partial z'} + \beta'_c c\right)\right] = \frac{bS^0}{n}(1+\beta'_c c)\frac{\partial h'}{\partial t'} + \beta'_c \frac{\partial c}{\partial t'} - \frac{q'}{K^0} \qquad (8)$$

$$(1+\beta'_c c)\left(\frac{\partial^2 c}{\partial x'^2} + \frac{\partial^2 c}{\partial z'^2}\right) = \frac{bK^0}{nD_m}\frac{\partial c}{\partial t'} + Q_x \frac{\partial c}{\partial x'} + Q_z \frac{\partial c}{\partial z'} \qquad (9)$$

From the non-dimensional governing equations (eq. 8 and 9), we note that the seawater intrusion problem depends from the following three non-dimensional parameters:

$$\alpha = \frac{q'}{K^0} \qquad \beta = \frac{bK^0}{nD_m} \qquad \alpha' = \frac{bS^0}{n} \qquad (10a,b,c)$$

where q' is the freshwater recharge rate (m/d), K^0 the freshwater hydraulic conductivity (m/d), b the aquifer thickness (m), n the porosity (-), D_m the molecular diffusion coefficient (m^2/d) and S^0 the freshwater specific storage (1/m).

We note that the hydraulic conductivity appears in two of the non-dimensional parameters, α and β.

4. Results

Based on the previous analysis, a number of numerical simulations were carried out up to the equilibrium state for different values of the non-dimensional parameters (eq. 10) and allow the construction of non-dimensional diagrams (Fig. 2,3 and 4) of salt distribution for a homogeneous and isotropic confined aquifer with horizontal base and constant thickness that is uniformly recharged with freshwater.

- By decreasing the value of parameter $\alpha=q'/K^0$ (i.e. for lower values of fresh water inflow or for greater values of hydraulic conductivity), seawater intrusion is advancing inland and the width of dispersion zone is becoming wider (Fig. 2 and 3).
- By increasing the parameter $\beta=bK^0/nD_m$, the seawater-freshwater transition zone is narrowing and shifted to the seaside at the upper part of the aquifer, while the intrusion of saltwater is advancing inland at the lower part of the aquifer (Fig. 4).
- The distribution of the salts in the aquifer was found essentially identical for different values of the parameter $\alpha'=bS^0/n$; hence this parameter exhibits very low sensitivity, which makes it of low importance, especially for real case studies.

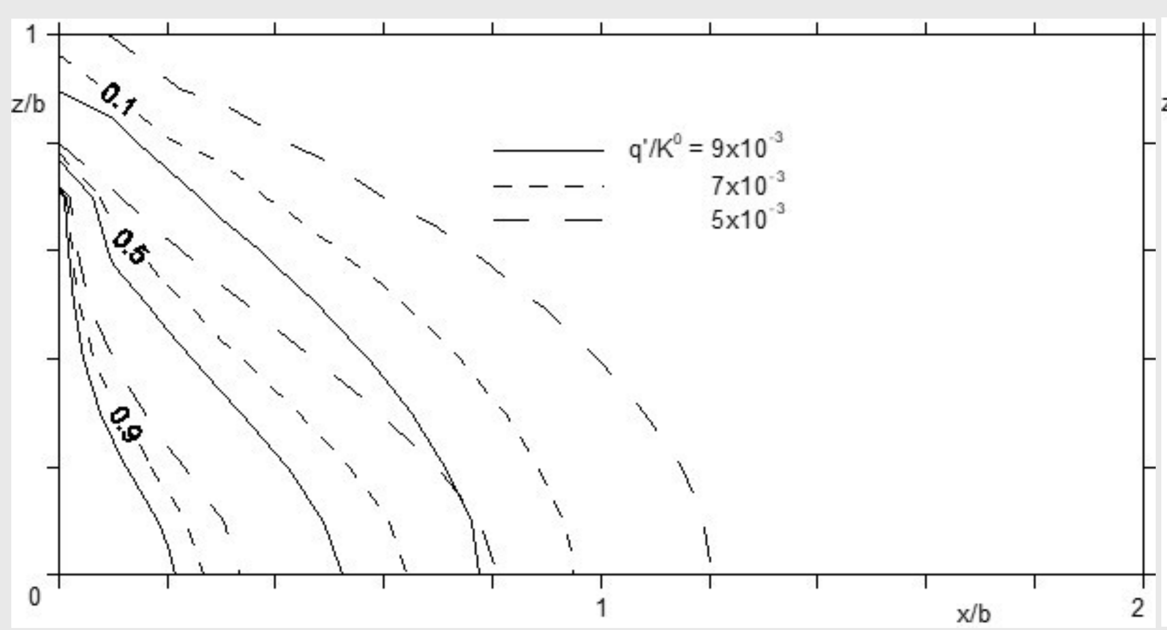

Fig.2. *Groundwater salinity distribution for $bK^0/nD_m = 2x10^3$*

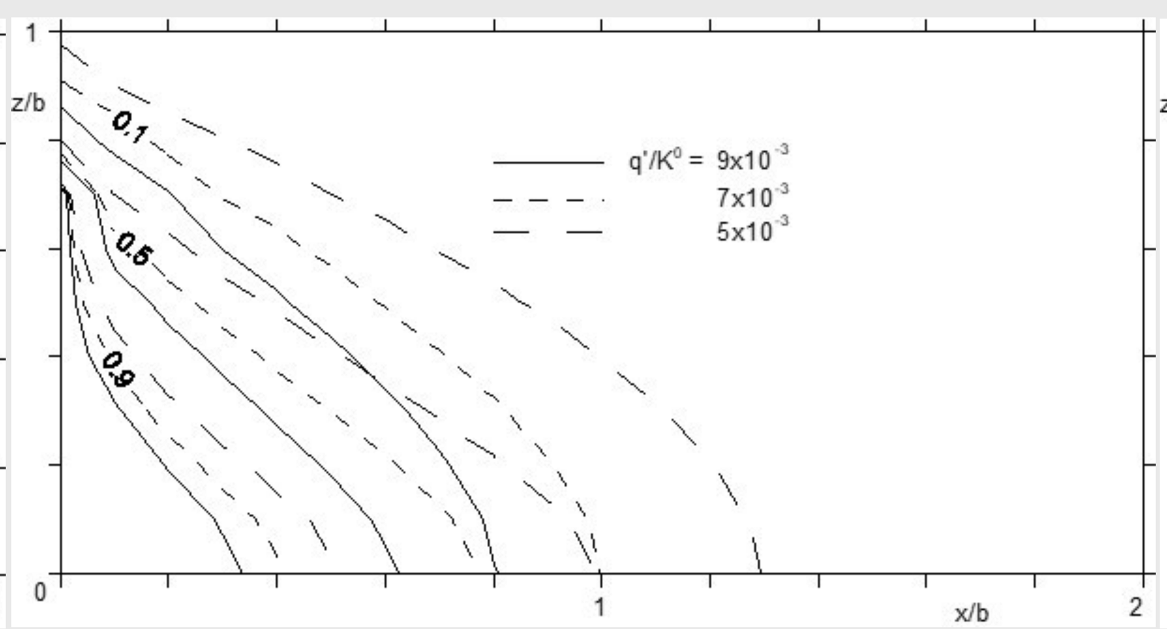

Fig.3. *Groundwater salinity distribution for $bK^0/nD_m = 4x10^3$*

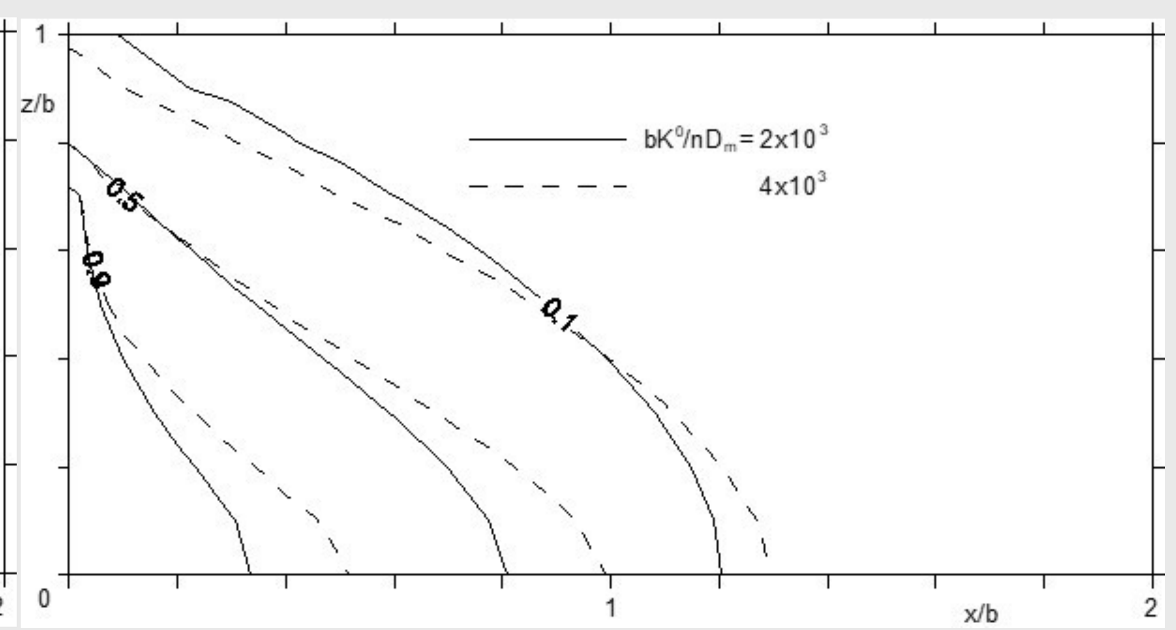

Fig.4. *Groundwater salinity distribution for $q'/K^0=5x10^{-3}$*

5. Conclusions

The non-dimensional diagrams – constructed by following the variable density flow approach and under specific assumptions – can be used for a quick and direct prediction of seawater intrusion in real aquifers. These diagrams would be useful for an initial prediction at the case studies of the ongoing PRIMA MEDSAL project (www.medsal.net), namely the coastal aquifers in Rhodope (Greece), Samos island (Greece), Bouficha (Tunisia), Bouteldja (Algeria), Tarsus (Turkey) and under specific assumptions to the karstic aquifer in Salento (Italy).

References

Doulgeris Ch., 2005. Seawater intrusion in coastal aquifers – Mathematical simulation (in Greek). School of Agriculture, Aristotle University of Thessaloniki, Ph.D. thesis.

Doulgeris, Ch. and Zissis, T., 2014. 3D variable density flow simulation to evaluate pumping schemes in coastal aquifers. Water Resources Management, 28, 4943-4956. http://dx.doi.org/10.1007/s11269-014-0766-0

Acknowledgements

This work is partly supported by MEDSAL Project © (www.medsal.net), which is part of the PRIMA Programme supported by the European Union's Horizon 2020 Research and Innovation Programme and funded by the national funding agencies of GSRT, MESRS/DGRSDT, BMBF, RPF, MIUR, MHESR and, TÜBITAK.